II. "On the Specific Gravity of Alloys." By A. MATTHIESSEN, Ph.D. Communicated by Professor WHEATSTONE. Received November 17, 1859.

This communication consists of a revised version of a Paper of the author, having the same title, which was read on the 19th May, and of which an abstract has been already given under that date (page 12).

III. "On an extended Form of the Index Symbol in the Calculus of Operations." By WILLIAM SPOTTISWOODE, Esq., M.A., F.R.S. Received November 24, 1859.

(Abstract.)

In the case of two variables (the only one considered in the present paper), the term Index Symbol means the operation

$$\nabla = x\frac{d}{dx} + y\frac{d}{dy}.$$

The new symbol is

$$\nabla_1 = y\frac{d}{dx} + x\frac{d}{dy}.$$

The symbols Ξ, Ξ_1 have the following meaning,

$$\Xi = x\frac{d'}{dx} + y\frac{d'}{dy},$$

$$\Xi = y\frac{d'}{dx} + x\frac{d'}{dy},$$

where the accent indicates that in the combinations of Ξ, Ξ_1 the differentiations are to affect the subject of operation alone, and not x or y, so far as they appear explicit in the values of Ξ, Ξ_1. The first object of the paper is to develope the relations between the combinations of Ξ, Ξ_1 and ∇, ∇_1; and it is found that

$$\Xi_1{}^i = \begin{vmatrix} \nabla_1 & i-1 & 0 & .. & 0 \\ \nabla & \nabla_1 & i-2 & .. & 0 \\ \nabla_1 & \nabla & \nabla_1 & .. & 0 \\ . & . & . & ... & . \\ \nabla & \nabla_1 & \nabla & .. & \nabla_1 \end{vmatrix}, \text{ or } = \begin{vmatrix} \nabla_1 & i-1 & 0 & .. & 0 \\ \nabla & \nabla_1 & i-2 & .. & 0 \\ \nabla_1 & \nabla & \nabla_1 & .. & 0 \\ . & . & . & ... & . \\ \nabla & \nabla & \nabla & .. & \nabla \end{vmatrix}$$

according as i is even or odd.

Again,—

$$(a, b, \ldots \between \Xi, \Xi_1)^n = (a_n \Xi + \beta \Xi_1) \ldots (a_2 \Xi + \beta_2 \Xi)(a_1 \Xi + \beta_1 \Xi)$$

$$= \left| \begin{array}{lll} (\nabla \nabla_1 \between a_n \beta_n) & (a_{n-1} \beta_{n-1}) & \ldots \\ (\nabla \nabla_1 \ldots \between a_n \beta_n \between (\nabla \nabla_1 \between a_{n-1} \beta_{n-1}) & & \ldots \\ & & \ldots \end{array} \right|$$

where $(\nabla \nabla_1 \nabla \between a_2 \beta_2 \between \times \between a_1 \beta_1) = (\nabla \nabla_1 \nabla \between a_2 \beta_2 \between a_1 \beta_1)$.

It is further shown that the effect of the operations ∇, ∇_1 on a given function, $u = \Sigma a_i x^{n-i} y^i$, may be represented by

$$F(\nabla, \nabla_1)\, u = \Sigma \left\{ F\left(u, (u-i+1)\, \epsilon^{-\frac{d}{di}} + (i+1)\, \epsilon^{\frac{d}{di}}\right) \right\} a_i x^{n-i} y^i;$$

and the case of $\dfrac{u}{F(\nabla, \nabla_1)}$ is examined in detail.

The value of

$$s_1 s_2 \ldots s_i \frac{d^i}{dx^{i-j}\, dy^j}$$

in terms of $s_1 \dfrac{d}{dx}$, $s_2 \dfrac{d}{dx}$, .. $s_1 \dfrac{d}{dy}$, $s_2 \dfrac{d}{dy}$, .. are calculated, (1) when s_1, s_2, .. are any linear functions of x, y, (2) when they are any functions whatever; and, in case (1), the effect of the above operation on a given function is determined.

IV. "Problem on the Divisibility of Numbers." By Francis Elefanti, Esq. Communicated by Arthur Cayley, Esq.

Received November 24, 1859.

Problem. To find a proceeding by which the divisibility of a proposed integer N by 7 or 13, or by both 7 and 13, may be determined through the same rule.

Solution. We can designate the number N by $abcd \ldots mn$, so that (a) be the first or highest, and (n) the last or lowest digit in it, therefore we may put

$$N = abcde \ldots mn.$$